Motorcycle Moments of Inertia and Angular Momenta

The total angular momentum of a motorcycle moving in a straight line comes principally from two sources: its engine and wheels.

Let's assume a traditionally mounted inline-four engine configuration with the crankshaft rotating in the same plane as the wheels. Assume that the mass of the crankshaft is about 12 kilograms, its radius is about 5 centimeters, its moment of inertia about its rotational axis is $\frac{1}{2}MR^2$ and that it rotates at 5000 rpm.

The approximate moment of inertia of the crankshaft is:

$$I_{cs} = \frac{1}{2}MR^2 = (0.5)(12kg)(0.05m)^2 = 0.02kg \cdot m^2$$

The angular speed, ω, of the crankshaft at 5000 rpm is:

$$\frac{5000rev}{min} \times \frac{1min}{60s} \times \frac{2\pi rad}{rev} = 523rad \cdot s^{-1}$$

The angular momentum, ℓ, of the crankshaft is:

$$\ell_{5,000rpm} = I\omega = (0.02kg \cdot m^2)(523s^{-1}) = 10.5kg \cdot m^2 \cdot s^{-1}$$

This estimate undervalues the contribution of the engine to the total angular momenta of the motorcycle by ignoring the other engine parts that rotate in the same plane of interest, e.g., the cam shaft, clutch, etc. A better estimate of the value of the total angular momentum supplied by the engine is probably about double this figure, i.e. $20kg \cdot m^2 \cdot s^{-1}$ for an engine spinning at 5000 rpm. For an engine spinning at 10,000 rpm this figure doubles to about $40kg \cdot m^2 \cdot s^{-1}$

Now let's examine the contribution of the wheels to the total angular momentum of the motorcycle. Assume that the mass of a 17 inch (43.2 cm) front motorcycle wheel (including the tire and brake rotors) is 15 kilograms and that the tire mounted on the wheel is a 120/70. This yields a radius of about 30 centimeters for the system.

The approximate moment of inertia of the wheel is MR^2, hence:

$$I_{wheel} = MR^2 = (15kg)(0.30m)^2 = 1.4kg \cdot m^2$$

At a speed of 20 m/s (about 45 mph), the angular speed of the wheel is:

$$\frac{20meters}{s} \times \frac{1rev}{2\pi rmeters} \times \frac{2\pi rad}{rev} = 67rad \cdot s^{-1}$$

The angular momentum, ℓ, of the wheel is:

$$\ell_{20\mathrm{m}\cdot\mathrm{s}^{-1}} = \mathrm{I}\omega = \left(1.4\mathrm{kg}\cdot\mathrm{m}^2\right)\left(67\mathrm{s}^{-1}\right) = 94\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}$$

In most cases the rear wheel will have reasonably similar values for mass and radius so the total angular momentum from both wheels at 20 m/s is about $188\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}$. If the speed doubles the angular momentum supplied by each wheel also doubles.

So the contribution to the total angular momentum of a motorcycle traveling in a straight line at 45 mph from an engine spinning at 5000 rpm is roughly 1/5 of what each wheel supplies. Note, however, that the front wheel itself supplies less than half of the total angular momentum of the entire system.